Application Note 1

Linear Phase 8th Order Elliptic Lowpass

Highlights
Multistage Analog Design
Elliptic Filters and Allpass Filters
Using the Curve Editor
Target Optimizer
Circuit Optimizer

■ Design Objective

Lowpass: -3dB@10kHz
Passband: ±0.1dB Flatness
Stopband: 80dB Min Atten @15kHz and above
Linear Phase response in the Passband

This Lowpass filter demands fairly high attenuation rates. Dropping the level 80dB from 10kHz to 15kHz is a slope rate of almost 160dB/Octave. Since the linear phase requirement is also a concern, we could first attempt to meet the design using Allpole filter families such as Linear Phase, Bessel, or Gaussain. These families provide flat group delay in the passbands, and would solve that issue.

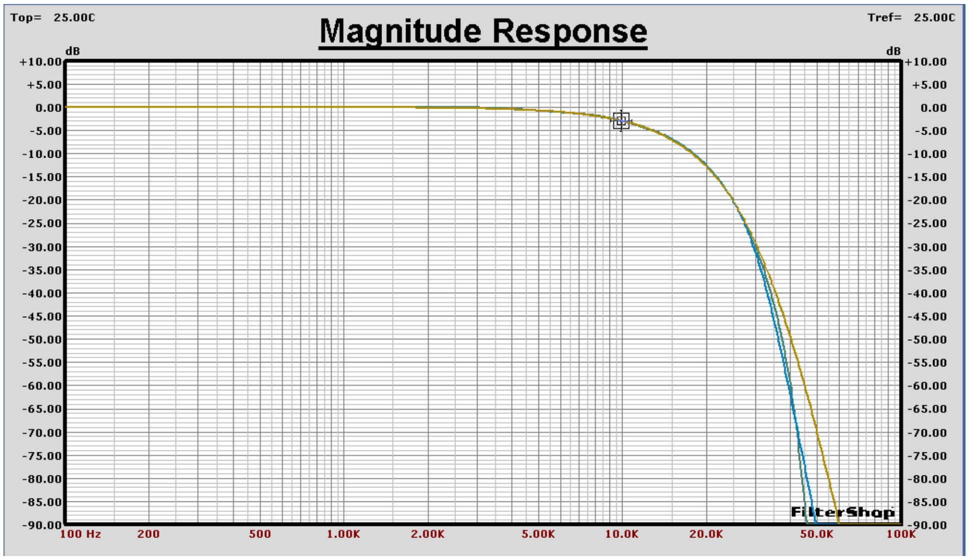
■ Scale Setup

Since the corner frequency is 10kHz, a suitable frequency range would be from 100Hz to 100kHz. This gives us a decade above and two below the transition frequency. A Log axis will be used, and an initial magnitude scale of 5dB/Div.

■ Allpole Filter Trials

To determine if any of the Allpole linear phase type filters can meet the attenuation requirements, we will first generate a 16th order Lowpass target of each type at the required 10kHz frequency. By using a custom 3dB transition level in each case, all of the filters will have the same -3dB@10kHz corner. Each of these were saved as Guide Curves. Data Curves with 500 points will be used.

The graph on the following page shows the results. None of the Allpole flat group delay Lowpass filters even come close to meeting the attenuation requirements, even at 16th order. The attenuation at 15kHz is only down about 10dB. No where near the 80dB requirement.



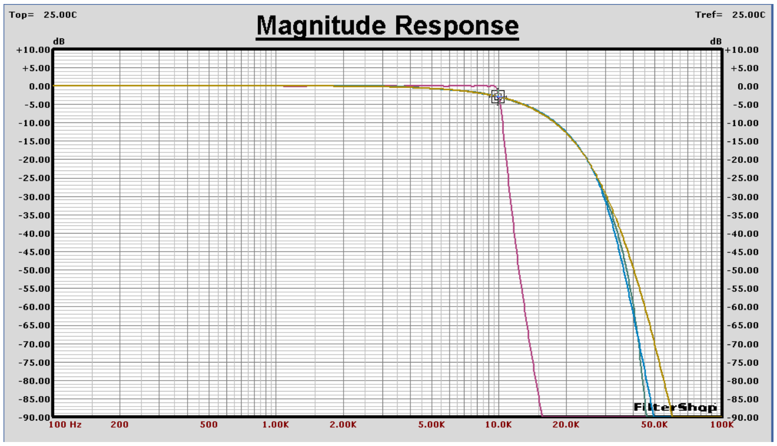
Since it will not be possible to meet the attenuation requirements using a standard flat group delay Allpole filter, a high attenuation type Lowpass must be used. However, this type of Lowpass filter will not exhibit flat group delay. To meet that requirement an Allpass filter must be designed to flatten the group delay.

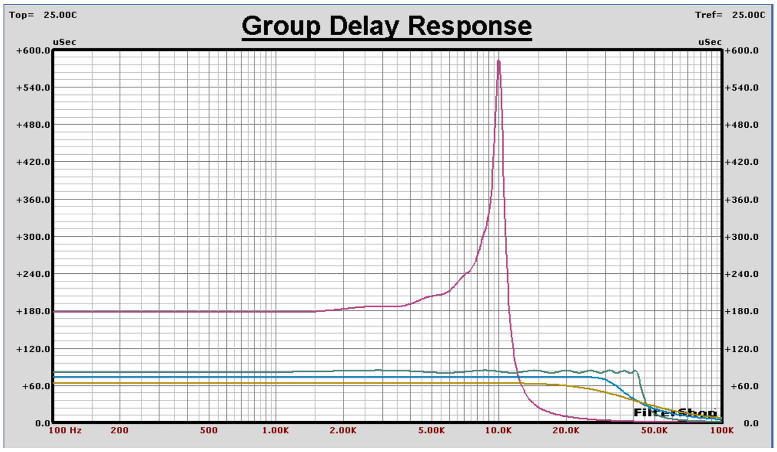
We can now attempt to use an Allpole filter with higher attenuation rates, such as a Chebyshev family. Standard Chebyshev filters demand very high precision components, and are generally difficult to produce accurately. The MCP form is much more practical.

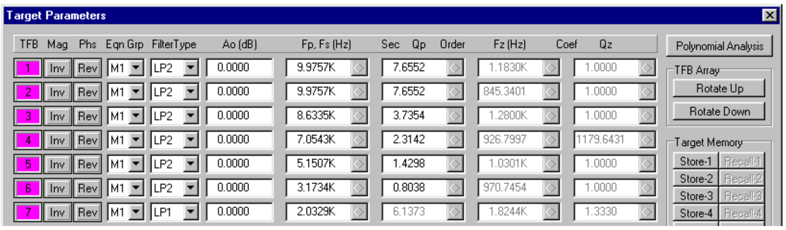
With a few trial & error attempts using different orders, it was found that a 0.1dB 13th order MCP/2 Chebyshev closely meets the requirements. The magnitude and delay response is shown on the following page.

This Lowpass provides more than 80dB@15kHz attenuation. The Group Delay graph shows the large peak that results from the Chebyshev family. A portion of the Target Parameters for the filter is also shown on the Group Delay graph. Note that the Qp value for the first two critical poles is 7.65. This would not be too hard to handle in a circuit realization.

Allpole MCP/2 Chebyshev, 0.1dB, 13th order







TFB	Mag	Phs	Eqn Grp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec Qp Order	Fz (Hz)	Coef Qz
1	Inv	Rev	M1	LP2	0.0000	9.9757K	7.6552	1.1830K	1.0000
2	Inv	Rev	M1	LP2	0.0000	9.9757K	7.6552	845.3401	1.0000
3	Inv	Rev	M1	LP2	0.0000	8.6335K	3.7354	1.2800K	1.0000
4	Inv	Rev	M1	LP2	0.0000	7.0543K	2.3142	926.7997	1179.6431
5	Inv	Rev	M1	LP2	0.0000	5.1507K	1.4298	1.0301K	1.0000
6	Inv	Rev	M1	LP2	0.0000	3.1734K	0.8038	970.7454	1.0000
7	Inv	Rev	M1	LP1	0.0000	2.0329K	6.1373	1.8244K	1.3330

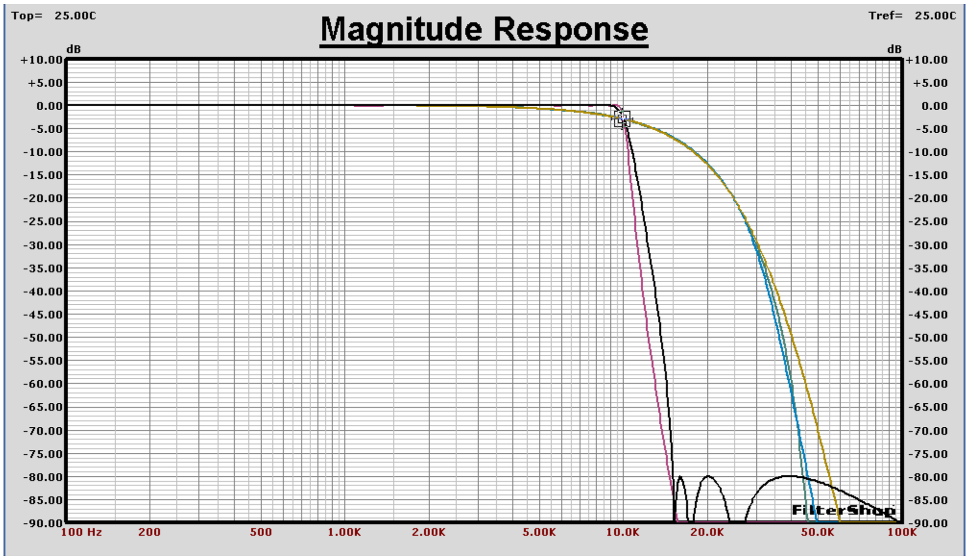
■ Elliptic Filter Trials

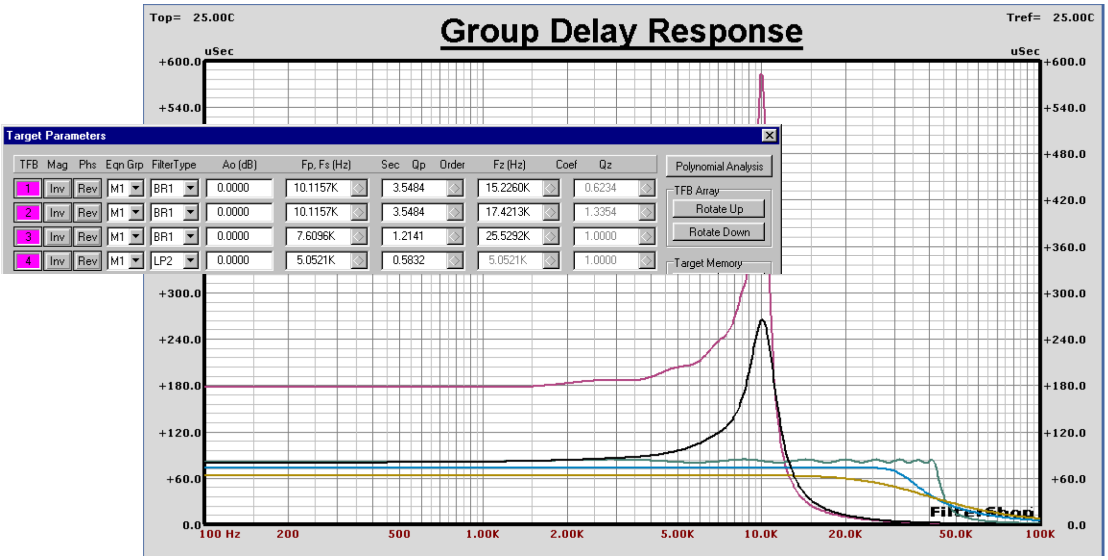
We have just seen that it took a 13th order Allpole filter to meet the attenuation requirements. The question now at hand is: what can an elliptic filter provide?

Using the MCP/2 Equal-Ripple elliptic family, several target attempts were made at different orders. It was found that an 8th order filter, with 0.05dB ripple and 80dB attenuation, met the goal of -80dB@15kHz. Again using a 3dB transition level.

The magnitude graph below shows the elliptic Lowpass target in Black, along with the previous Allpole attempts. On the next page, the Group Delay and Target Parameters are shown. The delay peak of the elliptic is much smaller than that of the previous MCP-Chebyshev Allpole, and the Qp value of the two critical poles is only 3.5.

The Elliptic filter meets the attenuation requirements at much lower order than the Allpole filter. Moreover, is has lower delay peaking and lower Q values as well. This includes a passband ripple of only 0.05dB, one-half of the design objective. This leaves some extra error margin.





Target Parameters

TFB	Mag	Phs	Eqn Grp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec Qp	Order	Fz (Hz)	Coef	Qz
1	Inv	Rev	M1	BR1	0.0000	10.1157K	3.5484		15.2260K		0.6234
2	Inv	Rev	M1	BR1	0.0000	10.1157K	3.5484		17.4213K		1.3354
3	Inv	Rev	M1	BR1	0.0000	7.6096K	1.2141		25.5292K		1.0000
4	Inv	Rev	M1	LP2	0.0000	5.0521K	0.5832		5.0521K		1.0000

The Elliptic filter is clearly the most efficient choice for this design, since it only requires 8th order vs. 13th order, and even has lower Q values. This will be our final choice for the Lowpass filter target.

■ Allpass Delay Compensation Network

The elliptic Lowpass filter does not have flat group delay. Rather, it has a peak near the corner frequency. To produce a complete Lowpass filter with linear phase across the entire passband, an Allpass network must be cascaded with the elliptic Lowpass filter. This Allpass network will increase the delay at lower frequencies, but does not affect the magnitude response (in theory).

The development of the Allpass compensation network proceeds as follows:

1 - Construct a Guide Curve for the desired Group Delay response.
2 - Choose an initial order for the Allpass network.
3 - Optimize the Allpass network F/Q parameters.

The required order of the Allpass network is unknown. There is no specific order that must be used. Often it is common to use an order equal to that of the filter being compensated. However, in most cases the desired flatness of group delay will determine the order required.

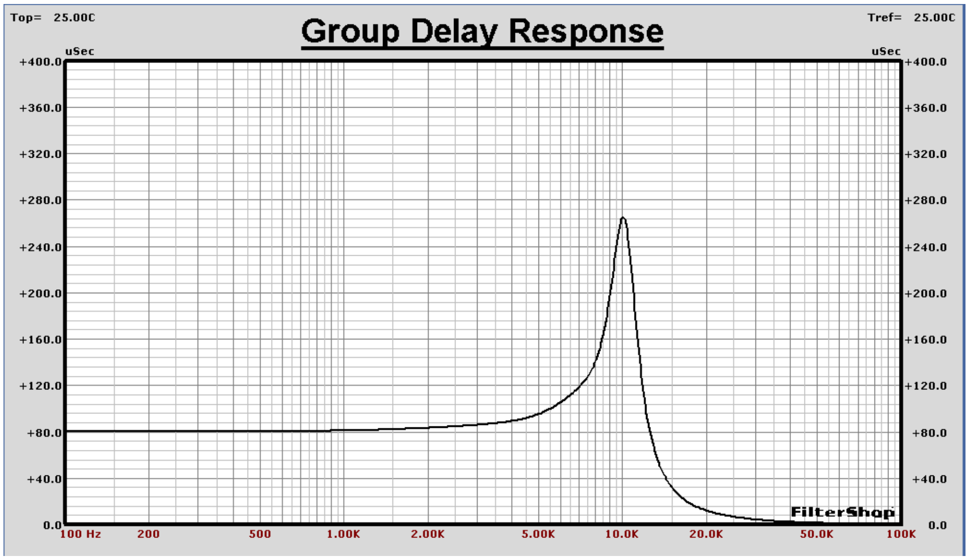
■ Defining the Group Delay Objective

Before we can optimize the Allpass network, we will need to manufacture an *objective* curve of our desired Group Delay response. The Target Optimizer will use this Guide Curve as the objective for curve optimization.

The Group Delay graph below now shows the elliptic Lowpass response by itself. The other previous Allpole curves have been disabled. At low frequencies the delay is 80uS, but increases by 350% to a peak of 270uS near 10kHz.

To produce a flat group delay, we will need to increase the delay to a constant value *higher* than the highest peak of the Lowpass alone. Therefore the objective is a *flat line* curve with a delay value greater than 270uS.

The *Curve Editor* utility will be used to create this curve. Another method would be to use a text editor to create a data file with suitable values, and then import this into the program.



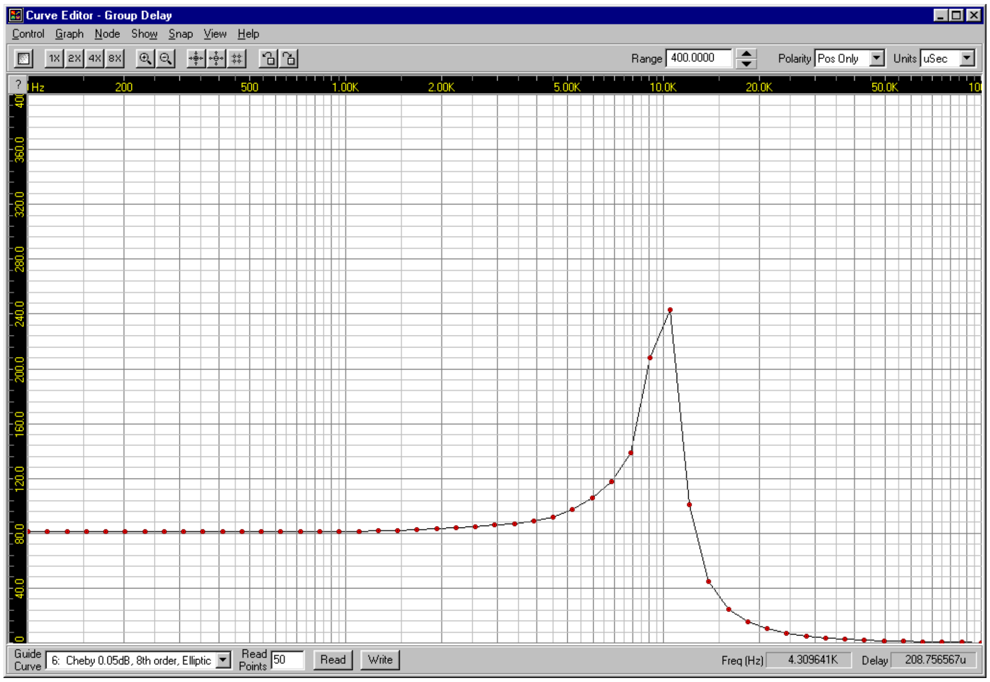
■ Creating the Objective with Curve Editor

Having created a copy of the elliptic Lowpass target in a Guide Curve, the Curve Editor is opened. Changing to the Group Delay graph, we first read in the points from the Lowpass Guide Curve. Since we are about to delete many of the points, only 50 data point density was used for reading the existing curve.

Now we wish to alter the curve in the passband to be a flat line. As an initial guess, we shall try a delay of 300uS. Since we only need a flat line, many of the nodes on the editing curve will be deleted in the passband region.

The content of the curve in the stopband region is unimportant. This area will not be part of the optimization.

The finished editor curve is shown on the next page. It is written to a different Guide Curve entry. The Group Delay graph at the bottom shows the Lowpass and Objective response as seen now in the main system.



Range 400.0000

Polarity Pos Only

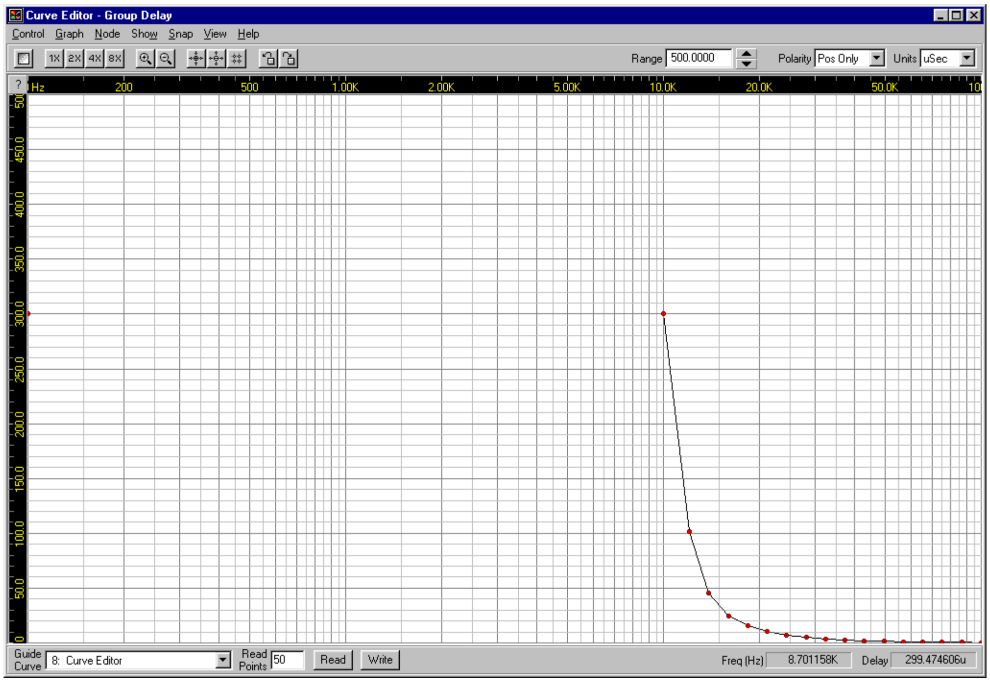
Units uSec

Guide Curve 6: Cheby 0.05dB, 8th order, Elliptic

Read Points 50

Freq (Hz) 4.309641K

Delay 208.756567u



Range 500.0000

Polarity Pos Only

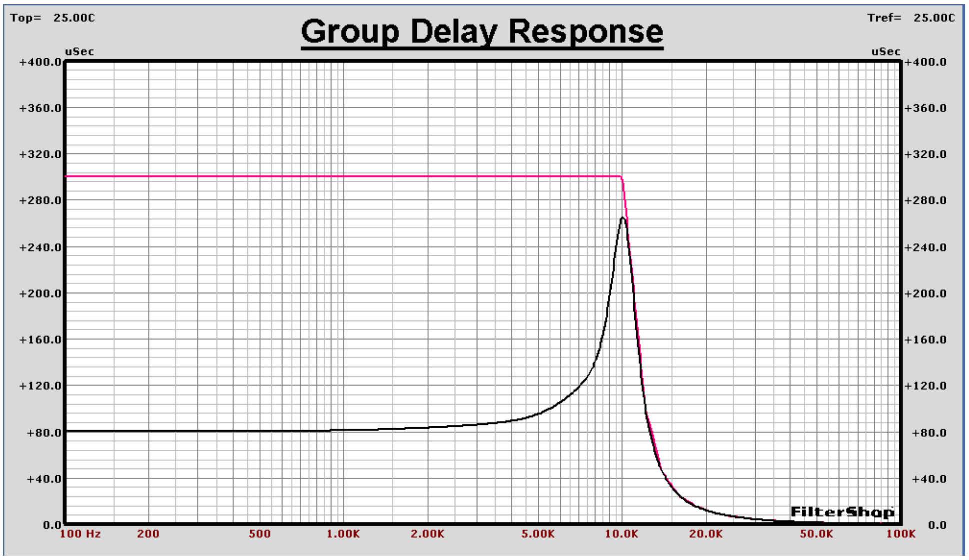
Units uSec

Guide Curve 8: Curve Editor

Read Points 50

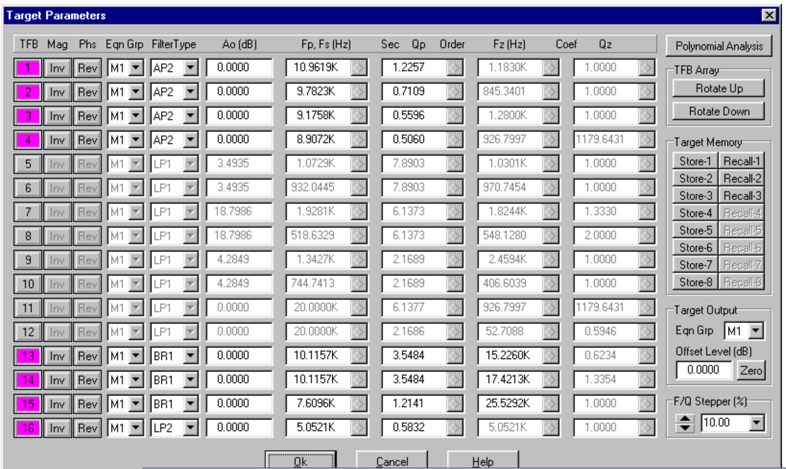
Freq (Hz) 8.701158K

Delay 299.474606u



■ Creating the Allpass Network

As an initial starting point we begin with an 8th order Allpass. We will use a standard Bessel Allpass with Fo of 5kHz to initialize the Target Parameters. Since we wish to cascade this with our existing Lowpass target, we will need to move the existing TFBs of the Lowpass filter down to the bottom of the TFB array, to preserve them from being overwritten.



TFB	Mag	Phs	Eqn Grp	FilterType	Ao (dB)	Fp, Fs (Hz)	Sec Qp Order	Fz (Hz) Coef	Qz
1	Inv	Rev	M1	AP2	0.0000	10.9619K	1.2257	1.1830K	1.0000
2	Inv	Rev	M1	AP2	0.0000	9.7823K	0.7109	845.3401	1.0000
3	Inv	Rev	M1	AP2	0.0000	9.1758K	0.5596	1.2800K	1.0000
4	Inv	Rev	M1	AP2	0.0000	8.9072K	0.5060	926.7997	1179.6431
5	Inv	Rev	M1	LP1	3.4935	1.0729K	7.8903	1.0301K	1.0000
6	Inv	Rev	M1	LP1	3.4935	932.0445	7.8903	970.7454	1.0000
7	Inv	Rev	M1	LP1	18.7986	1.9281K	6.1373	1.8244K	1.3330
8	Inv	Rev	M1	LP1	18.7986	518.6329	6.1373	548.1280	2.0000
9	Inv	Rev	M1	LP1	4.2849	1.3427K	2.1689	2.4594K	1.0000
10	Inv	Rev	M1	LP1	4.2849	744.7413	2.1689	406.6039	1.0000
11	Inv	Rev	M1	LP1	0.0000	20.0000K	6.1377	926.7997	1179.6431
12	Inv	Rev	M1	LP1	0.0000	20.0000K	2.1686	52.7088	0.5946
13	Inv	Rev	M1	BR1	0.0000	10.1157K	3.5484	15.2260K	0.6234
14	Inv	Rev	M1	BR1	0.0000	10.1157K	3.5484	17.4213K	1.3354
15	Inv	Rev	M1	BR1	0.0000	7.6096K	1.2141	25.5292K	1.0000
16	Inv	Rev	M1	LP2	0.0000	5.0521K	0.5832	5.0521K	1.0000

Target Output

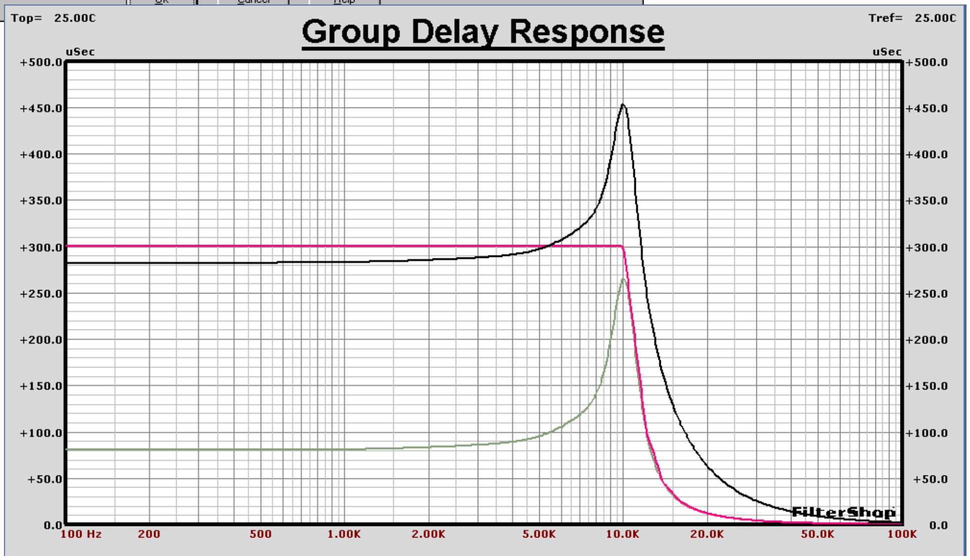
Eqn Grp M1

Offset Level (dB) 0.0000

F/Q Stepper (%) 10.00

After the Allpass target filter is run, we can go back to the Target Parameters and enable the four Lowpass TFBs at the bottom of the array, as shown here.

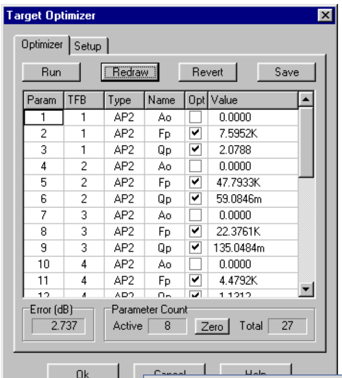
We now have a Lowpass elliptic filter and 8th order Allpass. The combined group delay is shown below.



■ Optimizing the Allpass Network

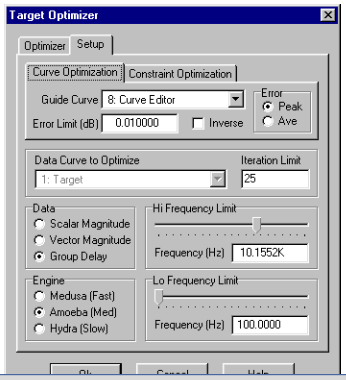
We now open the Target Optimizer, and setup the parameters as shown below. We use Guide Curve #8 which contains the objective curve, with Peak Error to produce an equal ripple result. The frequency span is limited to the passband from 100Hz to 10kHz. Only the eight Fp/Qp parameters of the four Allpass TFBs are selected for optimization.

After running the optimizer, the resulting delay response is shown below. The delay of the Lowpass/Allpass combination is now an equal ripple curve around the 300uS flat line objective. The linear phase ripple is about ±11%.



Param	TFB	Type	Name	Opt	Value
1	1	AP2	Ao	☐	0.0000
2	1	AP2	Fp	☑	7.5952K
3	1	AP2	Qp	☑	2.0788
4	2	AP2	Ao	☐	0.0000
5	2	AP2	Fp	☑	47.7933K
6	2	AP2	Qp	☑	59.0846m
7	3	AP2	Ao	☐	0.0000
8	3	AP2	Fp	☑	22.3761K
9	3	AP2	Qp	☑	135.0484m
10	4	AP2	Ao	☐	0.0000
11	4	AP2	Fp	☑	4.4792K

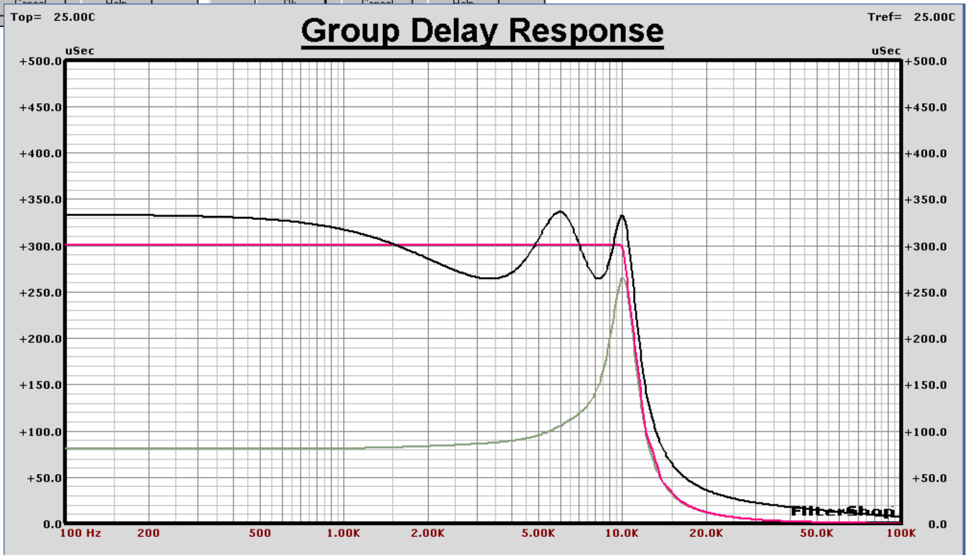
Error (dB): 2.737
Parameter Count: Active 8 Total 27



Guide Curve: 8: Curve Editor
Error Limit (dB): 0.010000
☐ Inverse
Error: ● Peak ○ Ave
Data Curve to Optimize: 1: Target
Iteration Limit: 25
Data: ○ Scalar Magnitude ○ Vector Magnitude ● Group Delay
Hi Frequency Limit: Frequency (Hz) 10.1552K
Engine: ○ Medusa (Fast) ● Amoeba (Med) ○ Hydra (Slow)
Lo Frequency Limit: Frequency (Hz) 100.0000

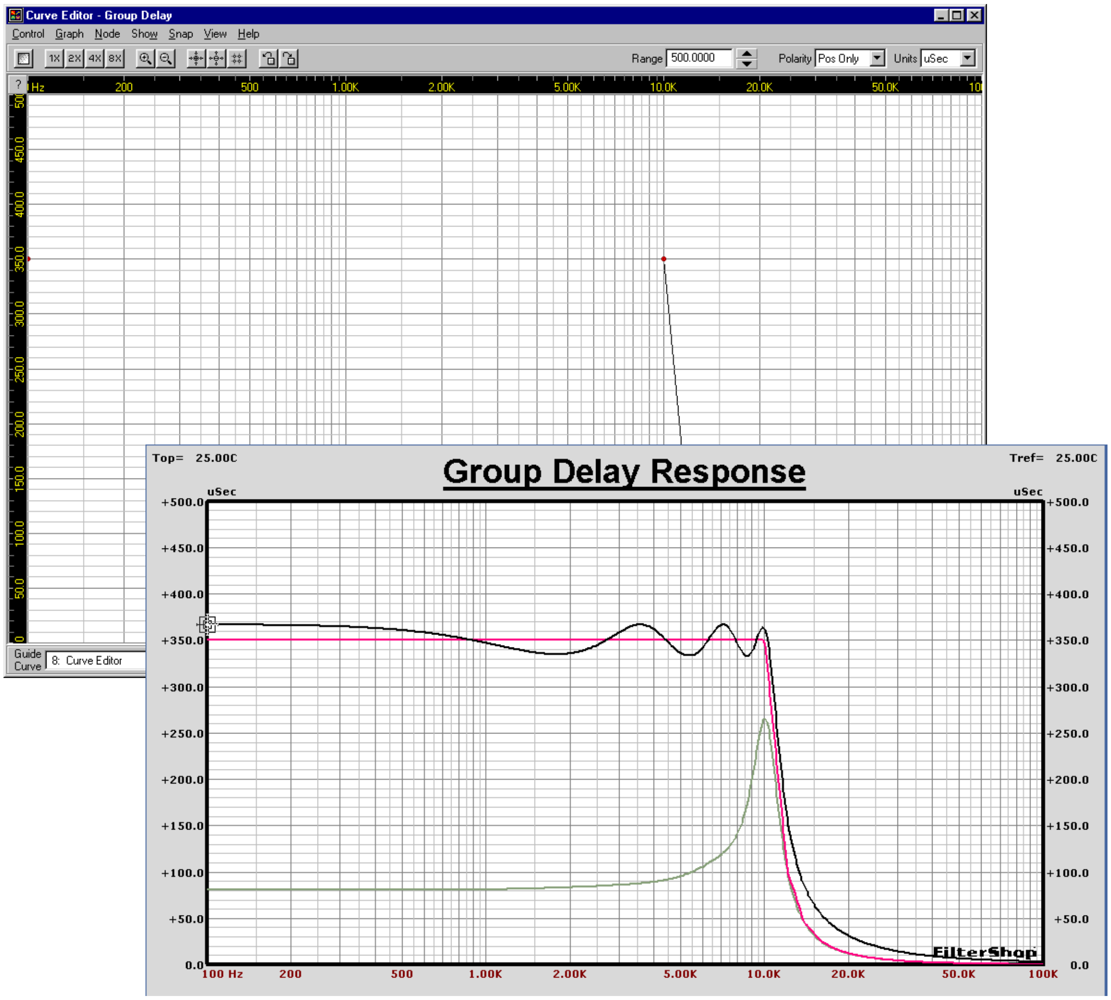
We chose the 300uS flat line objective as an initial guess. Using different values will result in different ripple levels.

From the response shown below, we see that most of the delay is above the 300uS line. Also, one of the Fp values of the AP2 sections ended up at a very high frequency, meaning that AP2 was not used.



We return to the Curve Editor, and this time move the flat line base level to a higher value of 350uS, as shown below. Guide Curve #8 is written again. The Target Optimizer is then run, optimizing to this new 350uS objective.

The Group Delay response, shown below, now has a reduced ripple of only ±5%. This is an improvement of over 200% from the previous. Also note that we have more ripples in the response, indicating all four AP2 sections are utilized.



Range 500.0000

Polarity Pos Only

Units uSec

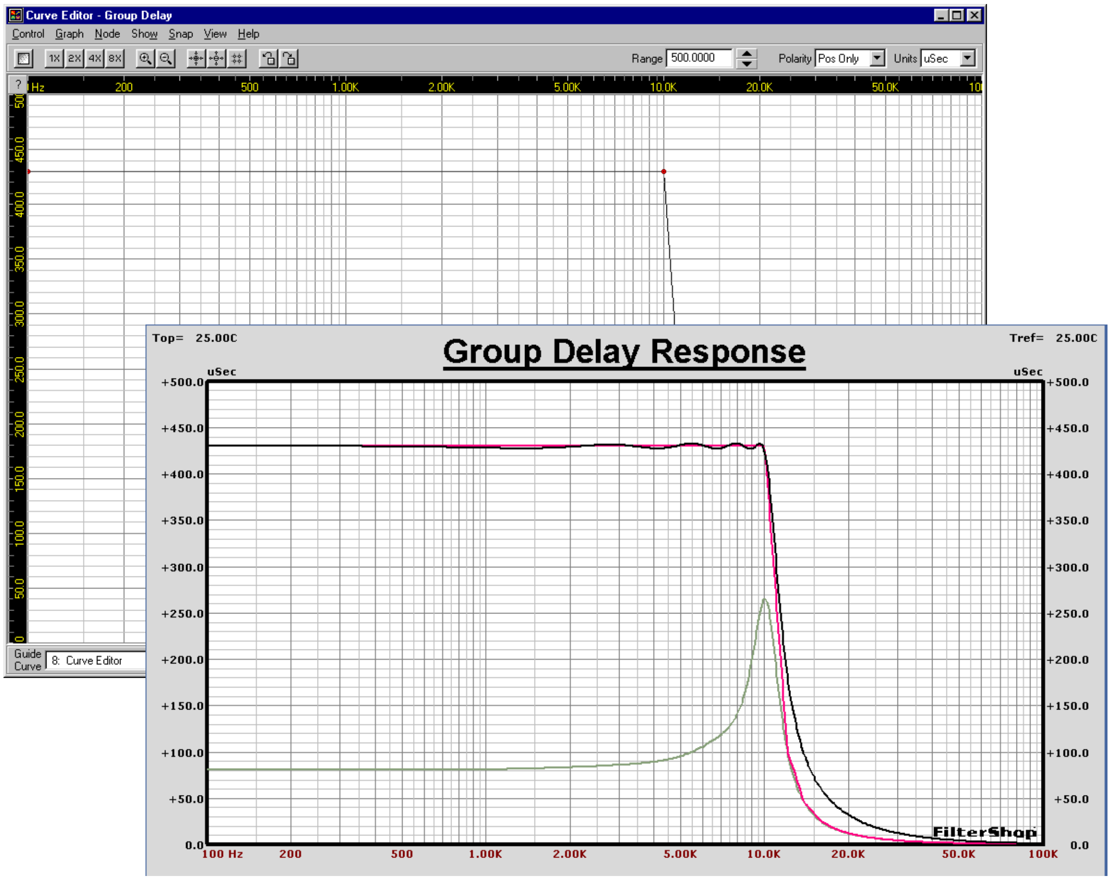
Guide Curve 8: Curve Editor

Top= 25.00C

Tref= 25.00C

Following this methodology, a couple more iterations show that the optimum base level of delay is near 430uS. This produces an equal ripple response of about ±0.7%, as shown in the graph below. This is an improvement of more than 700% from the previous, and is very flat.

If a higher base level of delay is attempted, the ripple increases again, with most of the delay below the objective. Therefore, using an 8th order Allpass network, the base line group delay is 430uS with a ripple of ±0.7%.



Range 500.0000

Polarity Pos Only

Units uSec

Guide Curve 8: Curve Editor

Top= 25.00C

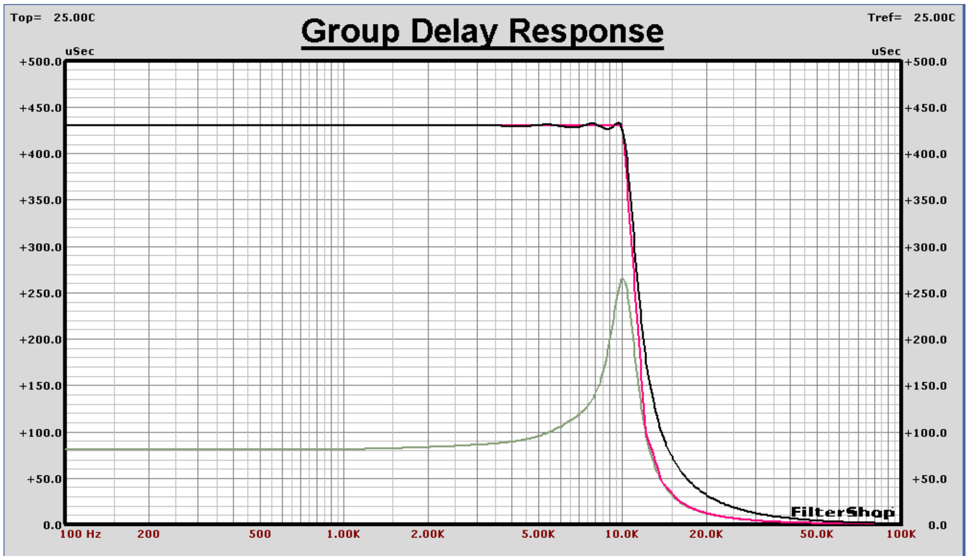
Tref= 25.00C

The previous optimizations were run using the *Peak Error* method. This produces an *equal-ripple* type optimization result. However, using the *Average Error* method we can obtain a *maximally-flat* type optimization result. This is shown below.

In this case the ripple is almost zero as frequency decreases, and is a maximum (±1%) concentrated near the 10kHz corner. Depending on your application or desire, either of these two results may be more or less attractive.

We can now consider what level of delay flatness could be obtained with *lower order* Allpass compensation networks. Suppose we try 6th and 4th order networks. These would require less circuitry, but would probably result in more ripple. Intuitively we would expect that the optimum base level of delay would occur at lower levels than the previous optimum for the 8th order case.

By disabling one or two AP2 TFB sections in the target, realigning the objective curve to lower levels , and reoptimizing the Allpass sections, the results shown on the next page were produced.

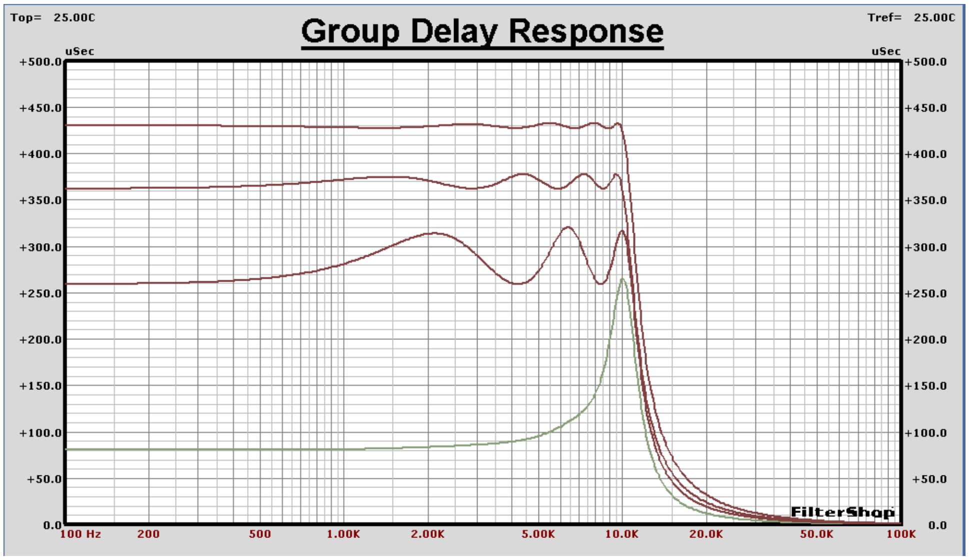


The graph below shows the optimum results for 8th, 6th, and 4th order Allpass compensation networks. They can be summarized as follows:

- 8th Order Allpass: 430uS, ±0.9% ripple
- 6th Order Allpass: 370uS, ±2.7% ripple
- 4th Order Allpass: 290uS, ±10% ripple

Choosing which configuration is best depends on what the criteria is for the delay characteristics. If we desire a minimum base level delay and/or minimum circuitry, then the 4th order option would be selected. If we want the best possible flatness, then the 8th order option would be selected. Higher order Allpass designs could further reduce ripple.

For this example, we will assume that a 2.7% ripple is satisfactory, and choose the 6th order Allpass design.



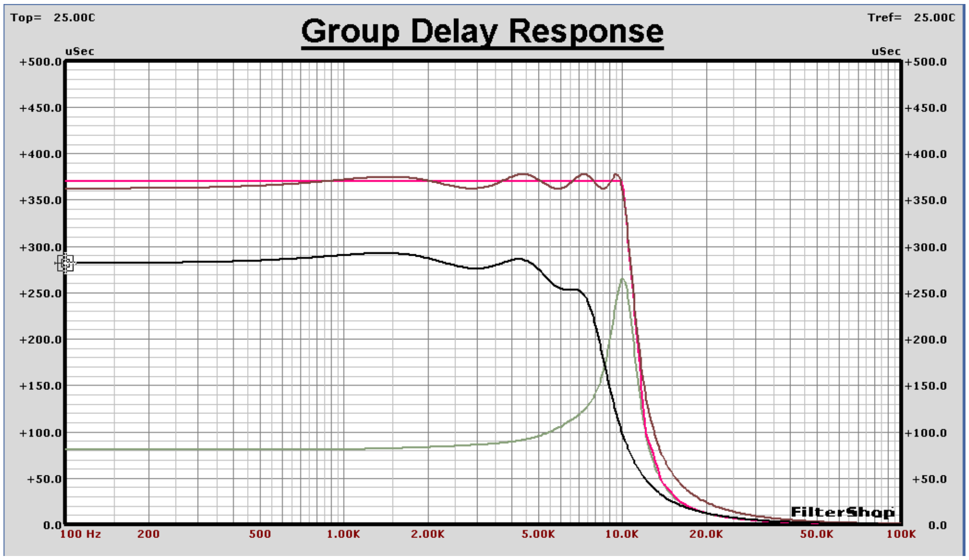
■ Final Target Design Summary

We now have the final target parameters for the complete design. An 8th order elliptic Lowpass will be cascaded with a 6th order Allpass.

8th Order Lowpass Elliptic:
BR1: Fp=10.1157kHz, Qp=3.5484, Fz=15.2260kHz
BR1: Fp=10.1157kHz, Qp=3.5484, Fz=17.4213kHz
BR1: Fp=7.6096kHz, Qp=1.2141, Fz=25.5292kHz
LP2: Fp=5.0521kHz, Qp=0.5832

6th Order Allpass:
AP2: Fp=7.7927kHz, Qp=2.0264
AP2: Fp=4.9260kHz, Qp=1.2204
AP2: Fp=2.5980kHz, Qp=0.6076

The graph below displays the group delay of the Lowpass and Allpass sections, along with their combined delay and objective Guide Curve.



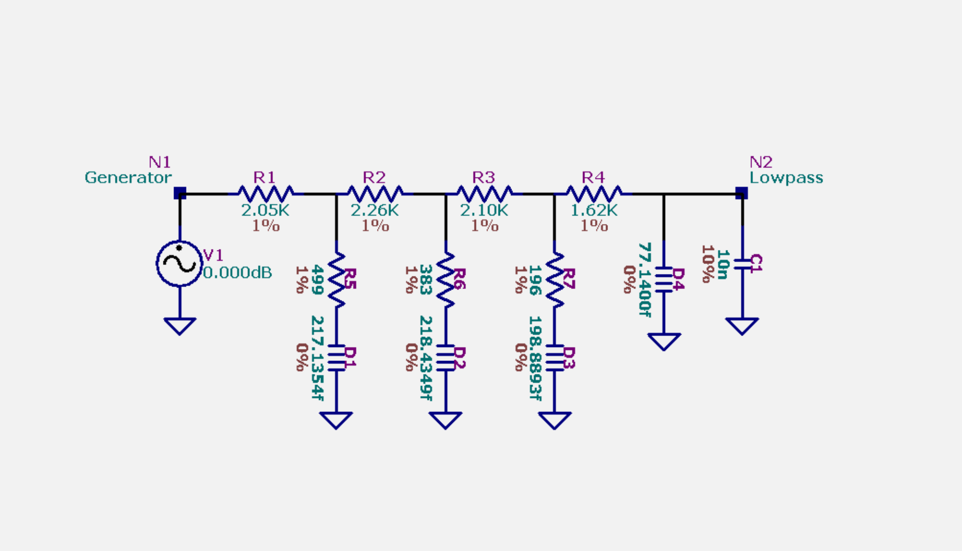
■ Lowpass Circuit Design

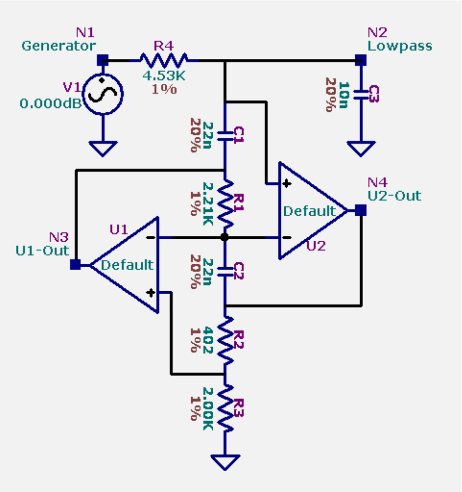
One of the best circuit topologies to handle this type of elliptic design is the RDC gyrator ladder. The LPE08_RDC synthesis circuit will be used. Choosing a preset value for R1=2K, the design shown below is produced.

The ideal four D values are:

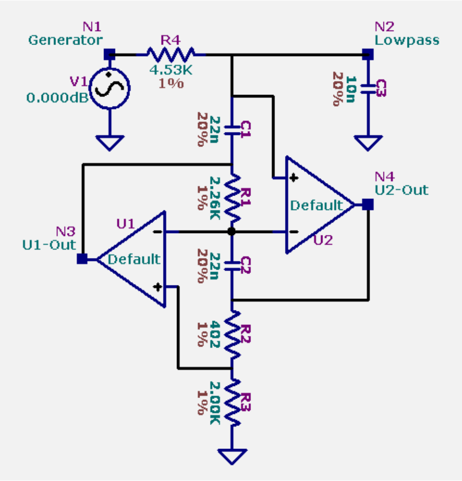
D1 = 217.1354f
D2 = 218.4349f
D3 = 198.8893f
D4 = 77.1400f

To realize each of the FDNR D elements, the LP2_GIC_D circuit is used to design each of the gyrators. Each of the GIC stages is shown on the following page.



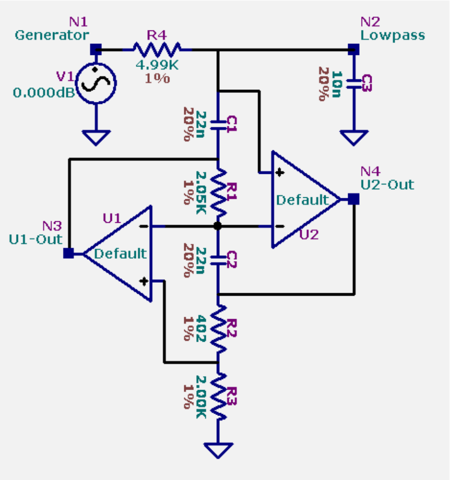


D1 Gyrator

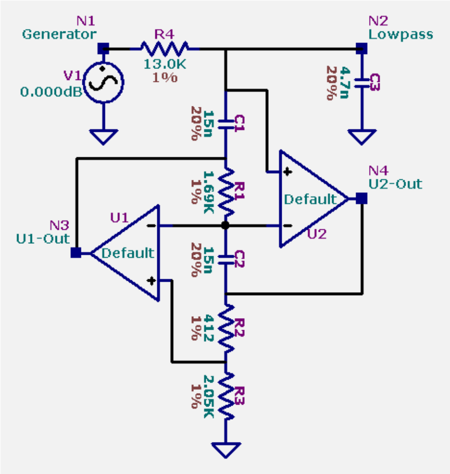


D2 Gyrator

D3 Gyrator

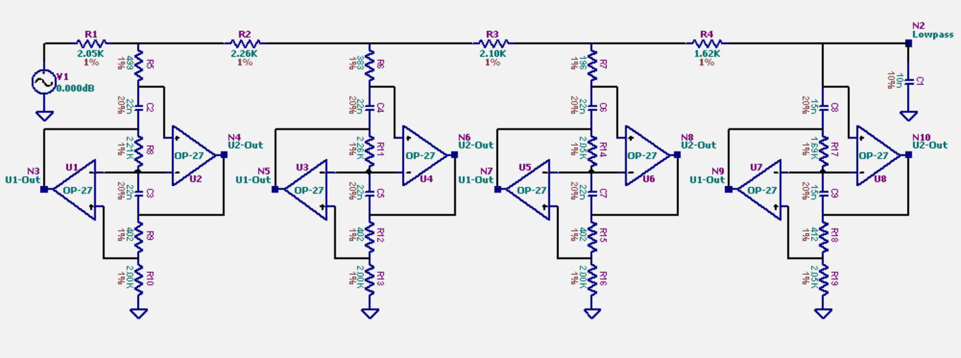


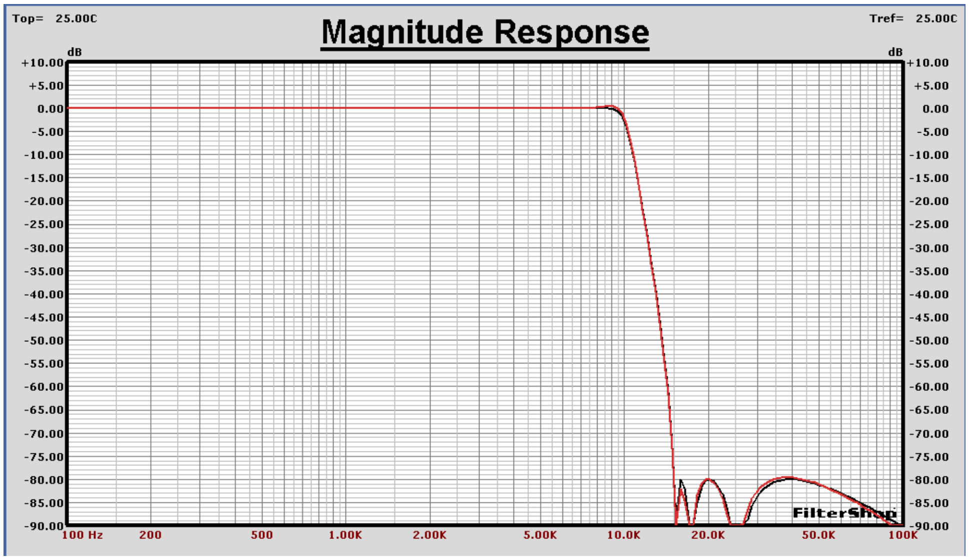
D4 Gyrator



The gyrator sections are now imported into the main RDC circuit, and used to replace the D elements. The extraneous components from the LP2_GIC_D circuits are also removed from each section. The resulting LPE8 circuit schematic is shown below, as well as the magnitude response.

The magnitude response of the circuit has a bit more ripple in the passband then the target. But as we shall soon see, the magnitude errors from the All-pass network are a larger problem.





■ Allpass Circuit Design

The 6th order Allpass network will be constructed by cascading three 2nd order sections. The AP2_RC_1 synthesis circuit will be used, which offers a worst case component sensitivity of about 2.

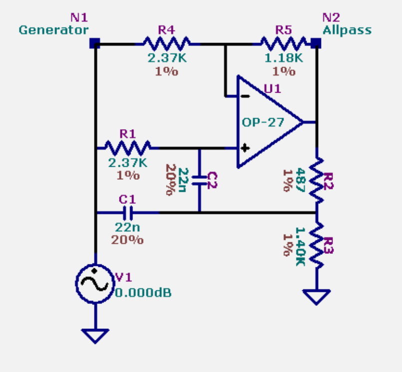
Each of the AP2 TFBs are enabled individually one at a time, and then synthesis run to design each section. Preset resistance values around 3k are used. After each stage is designed it is saved as a different file: AP2_1, AP2_2, and AP2_3. The three stages are shown here.

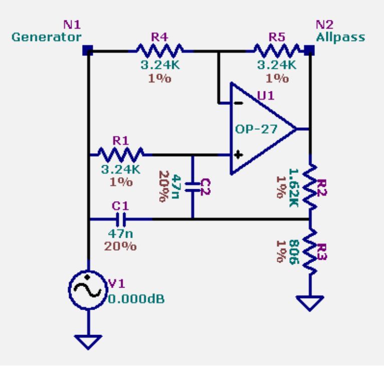
The stages are then combined into a single network by removing the extraneous components, and cascading the three sections. The final 6th order Allpass circuit is shown on the next page.

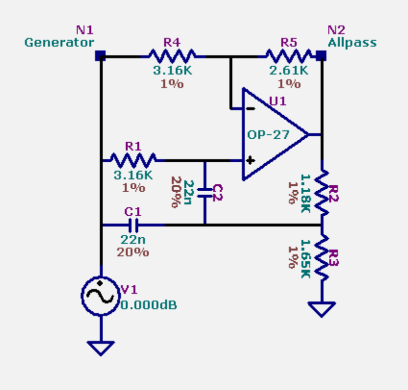
The Magnitude and Group Delay response curves for the Allpass network are shown on the next page as well. The delay curve appears as expected, however note that the Allpass magnitude is not perfectly flat. A broad rise of 0.5dB appears at 5kHz.

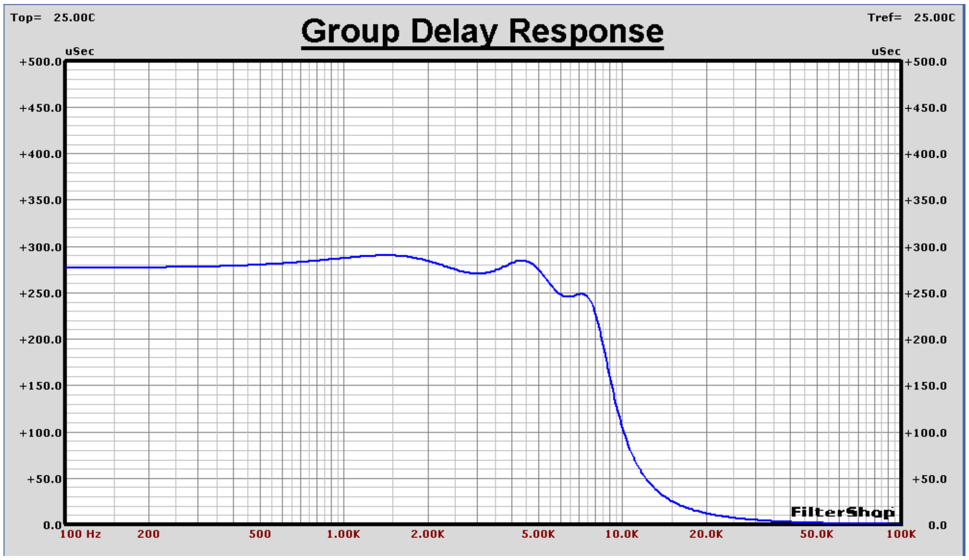
This is a common problem with Allpass filters. Due to finite GBW in the opamps and non-ideal component values, the magnitude response is not perfectly flat.

It will be necessary to adjust the combined Lowpass/Allpass filter to restore the passband magnitude flatness.





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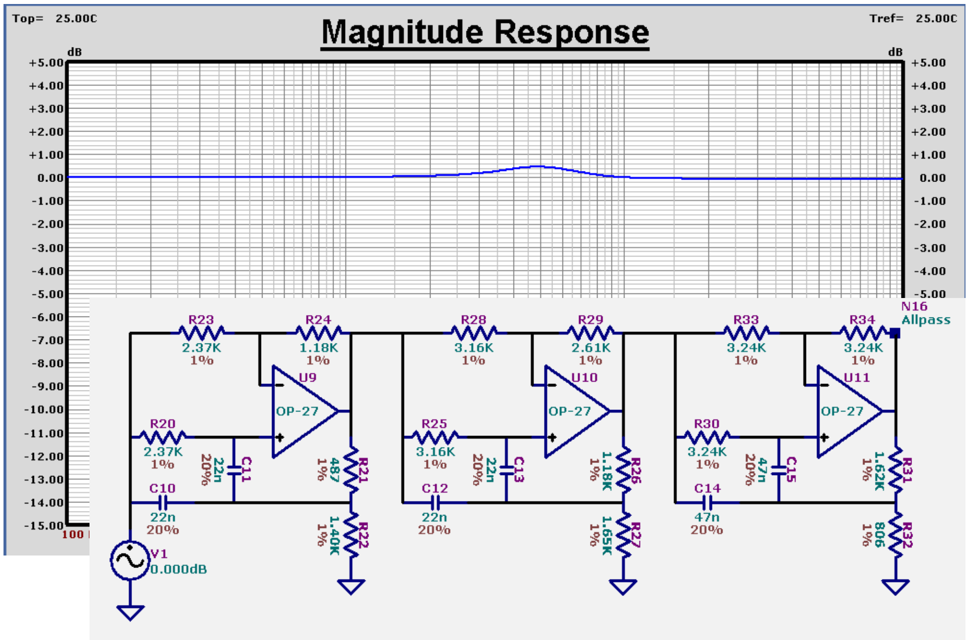
Group Delay Response
Tref= 25.00C
uSec
+500.0
+450.0
+400.0
+350.0
+300.0
+250.0
+200.0
+150.0
+100.0
+50.0
0.0
100 Hz
200
500
1.00K
2.00K
5.00K
10.0K
20.0K
50.0K
100K
FilterShop

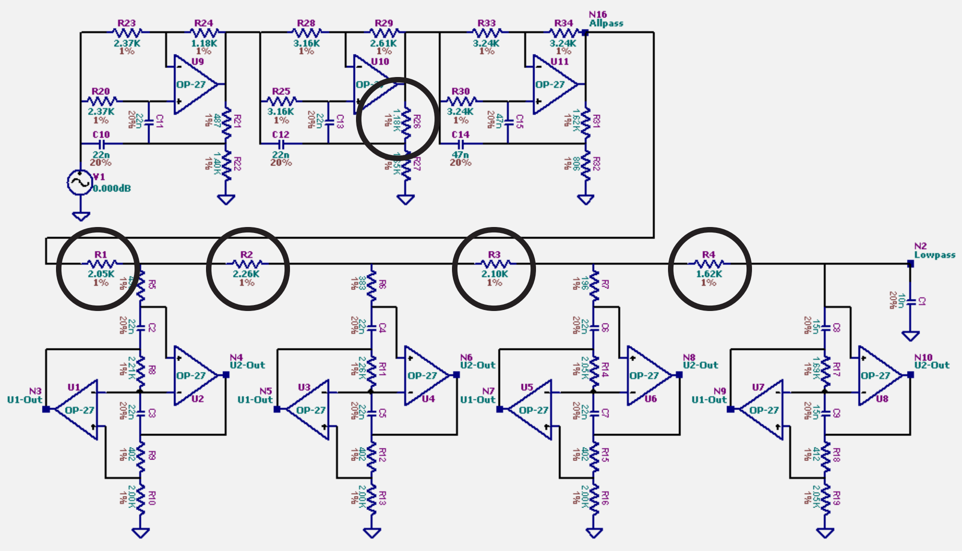

Top= 25.00C
Magnitude Response
Tref= 25.00C
dB
+5.00
+4.00
+3.00
+2.00
+1.00
0.00
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
-13.00
-14.00
-15.00
100
R23 2.37K 1%
R24 1.18K 1%
U9 OP-27
R20 2.37K 1%
C11 22n 20%
R21 487 1%
C10 22n 20%
R22 1.40K 1%
V1 0.000dB
R28 3.16K 1%
R29 2.61K 1%
U10 OP-27
R25 3.16K 1%
C13 22n 20%
R26 1.18K 1%
C12 22n 20%
R27 1.65K 1%
R33 3.24K 1%
R34 3.24K 1%
U11 OP-27
R30 3.24K 1%
C15 47n 20%
R31 1.62K 1%
C14 47n 20%
R32 806 1%
N16 Allpass

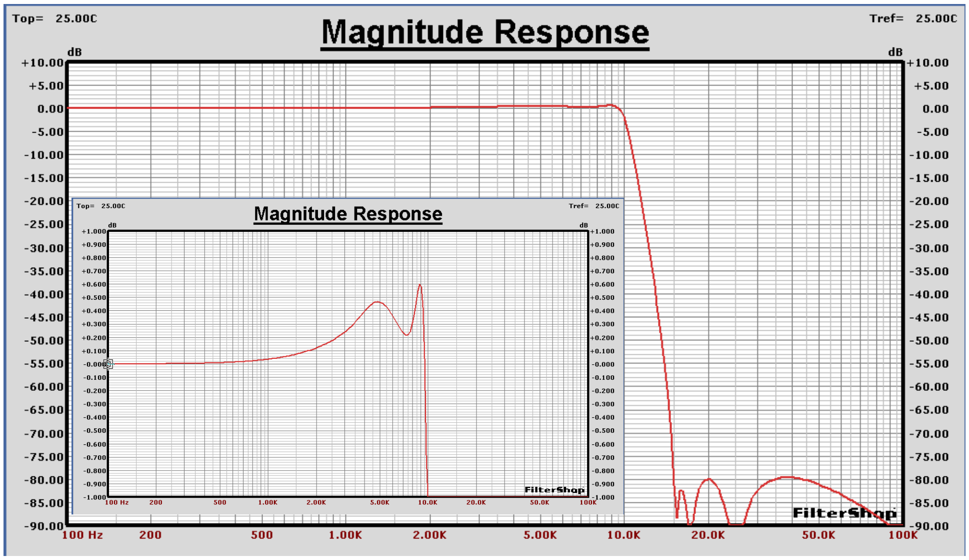
■ Correcting the Magnitude Errors

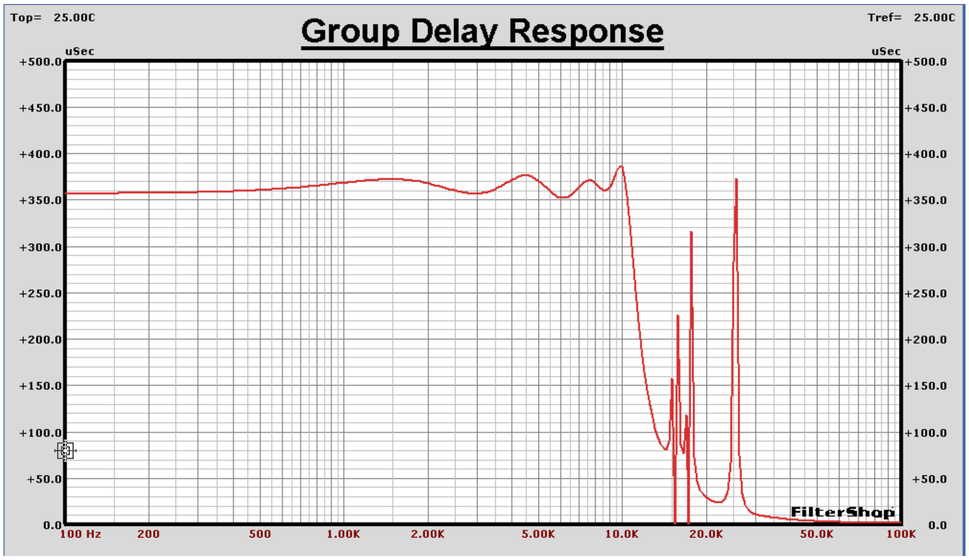
The combined Lowpass/Allpass circuit is shown below, and the magnitude/delay responses are shown on the next page. The group delay is very close to the original target. The stopband of the magnitude response also meets the design criteria, but the passband has excess ripple. The enlarged view of the passband shows a peak of about +0.6dB.

It was found that most of the Allpass network magnitude error was coming from the middle stage. R26 was adjusted to the next higher 1% value, which corrected most of the Allpass magnitude error.

For the remaining passband ripple error, we must adjust components in the Lowpass network. Since the group delay and stopband response are already acceptable, we need to choose components which will predominantly control the passband magnitude. The main resistors of the RDC topology R1, R2, R3, and R4 are probably good choices. They are circled below.



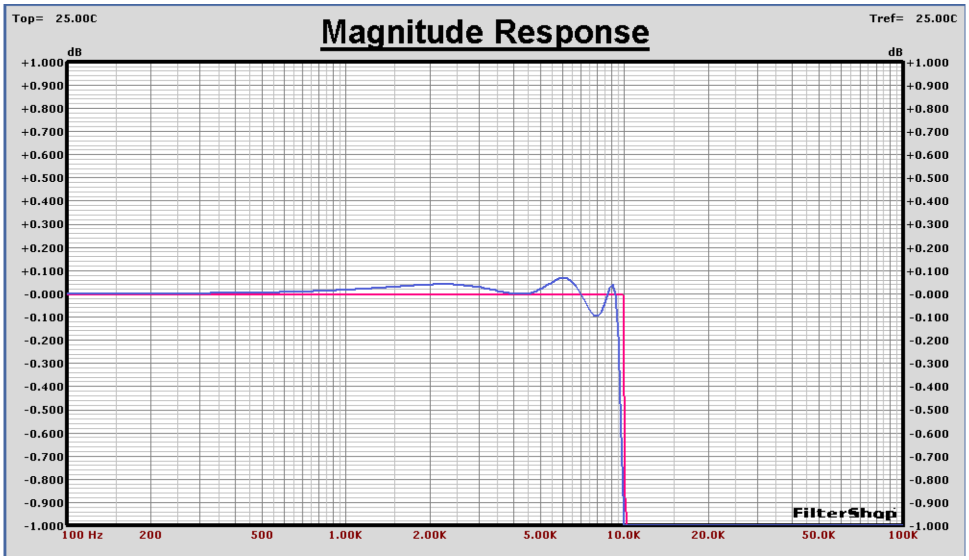

Top= 25.00C
Magnitude Response
Tref= 25.00C
dB
+10.00
+5.00
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
-45.00
-50.00
-55.00
-60.00
-65.00
-70.00
-75.00
-80.00
-85.00
-90.00
100 Hz
200
500
1.00K
2.00K
5.00K
10.0K
20.0K
50.0K
100K
FilterShop
Magnitude Response
+1.000
+0.900
+0.800
+0.700
+0.600
+0.500
+0.400
+0.300
+0.200
+0.100
-0.000
-0.100
-0.200
-0.300
-0.400
-0.500
-0.600
-0.700
-0.800
-0.900
-1.000


Top= 25.00C
Group Delay Response
Tref= 25.00C
uSec
+500.0
+450.0
+400.0
+350.0
+300.0
+250.0
+200.0
+150.0
+100.0
+50.0
0.0
100 Hz
200
500
1.00K
2.00K
5.00K
10.0K
20.0K
50.0K
100K
FilterShop

Since there are only four components to adjust, and the adjustment needed is very small, we could probably obtain the desired improvement using small 1% trial& error increments up/down in their values. However for this example an objective flat line magnitude curve at 0dB was created in Guide Curve #8 using the Curve Editor utility. The circuit was then scalar optimized on the magnitude response.

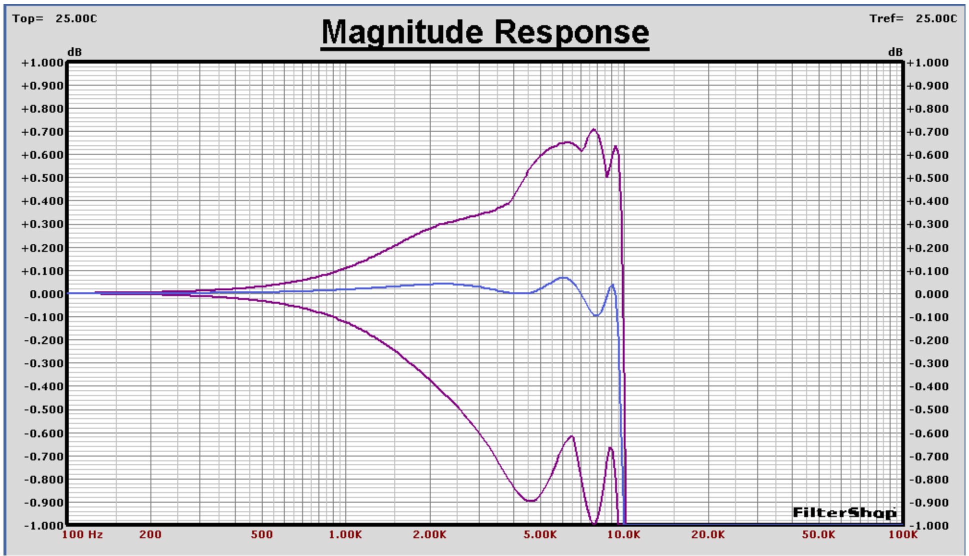
Since the response is -3dB@10kHz, we cannot run the optimization all the way to 10kHz. A frequency span of 100Hz to 9kHz was used.

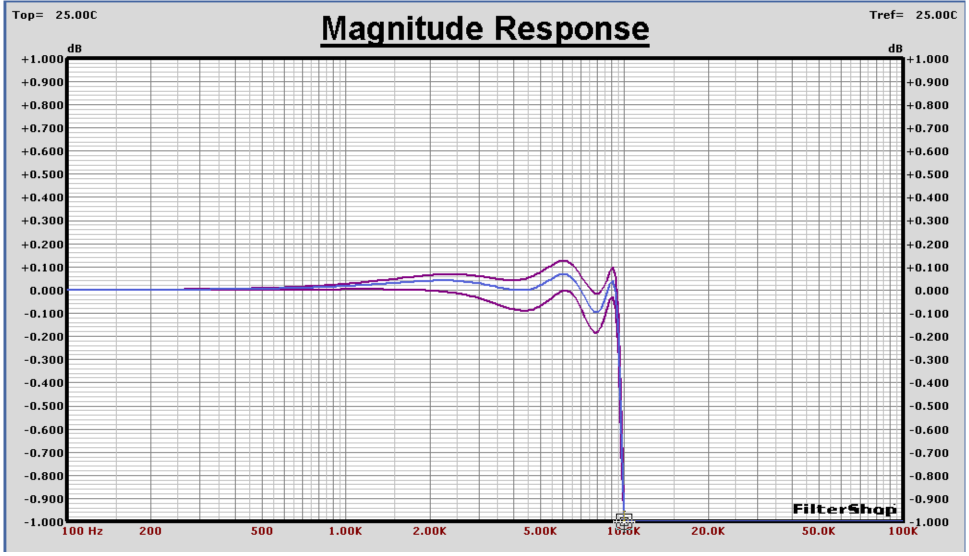
After the optimization was completed, the enlarged view of the passband is shown below. The response is now within the 0.1dB tolerance. Neither the stopband or group delay was significantly affected.



■ Monte Carlo Analysis

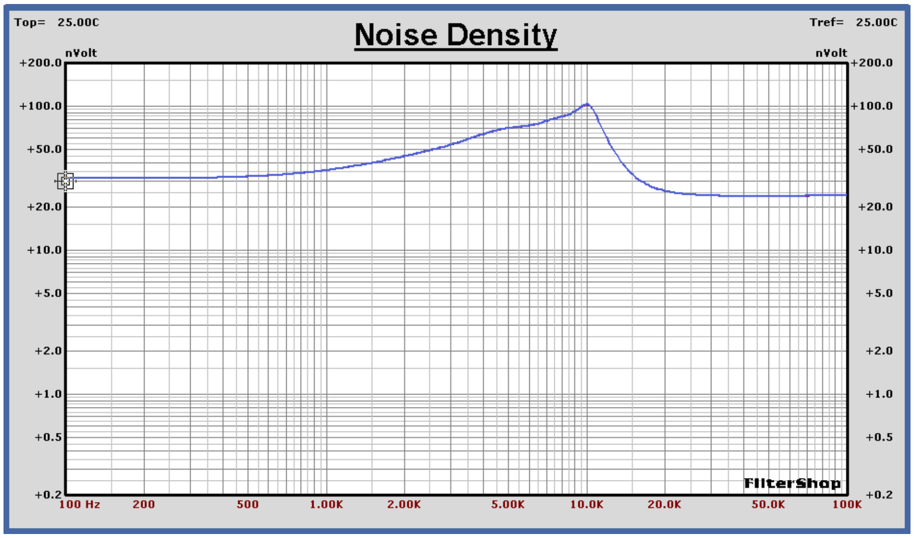
The two graphs below show the passband response variations for all components using 1% precision, or 0.1% precision. In order to maintain the 0.1dB passband window, 0.1% components are needed for many components.

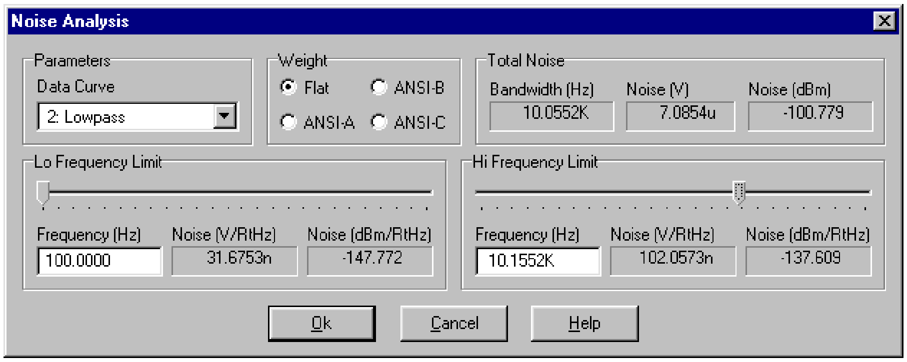




■ Noise Analysis

To gain a realistic idea of the noise performance of the circuit, all opamps were assigned to the OP-27 model. The noise density for the main output is shown below, along with the broadband Noise Analysis. For the passband width of 100Hz to 10kHz the total noise is -100.8 dBm, unweighted.
This completes the design.




Noise Analysis

Parameters
Data Curve: 2: Lowpass

Weight
● Flat ○ ANSI-B
○ ANSI-A ○ ANSI-C

Total Noise

Bandwidth (Hz)	Noise (V)	Noise (dBm)
10.0552K	7.0854u	-100.779

Lo Frequency Limit

Frequency (Hz)	Noise (V/RtHz)	Noise (dBm/RtHz)
100.0000	31.6753n	-147.772

Hi Frequency Limit

Frequency (Hz)	Noise (V/RtHz)	Noise (dBm/RtHz)
10.1552K	102.0573n	-137.609